**Unit 2 – Forces in 2D (Instructor Narrative)**

Engineering Statics in Physics Project

Carnegie Mellon University and Pittsburgh Area Schools

Paul S. Steif, Carnegie Mellon University

Janet R. Waldeck, Pittsburgh Allderdice High School

© July 11, 2013 Paul S. Steif and Janet R. Waldeck. All rights reserved.

**2.1 Forces can change the direction of motion**

Vectors are defined as having magnitude and direction. A mallet will be used to apply a force to a rolling bowling ball. This force will change the direction of the ball’s velocity.

LEARNING OBJECTIVE: Students will be able to associate an applied force with the observed change in velocity of a moving body.

**Class Demonstration and Discussion**

*Carefully* roll a bowling ball at a constant speed down the hallway. Apply a tap from a mallet to get the ball to make a turn while the ball is in motion:

On the diagram above, draw an arrow to indicate the force (size and direction) needed to make the bowling ball turn as shown. Is it possible for the bowling ball to change directions spontaneously (*i.e.* without any assistance from an externally applied force)?

*A change of direction, just like a change in speed, requires a force*

If the bowling ball is at rest on the floor we could push it in any direction

On the xy-axis shown below, draw an arrow to represent the force you used to change the direction of the moving bowling ball. Place the tail of the arrow at the origin and label “direction” in terms of the angle relative to the positive x-axis.

y

x

*SEE STUDENT WORKSHEET*

**2.2 Forces are vectors**

Spring scales attached to a central ring will provide physical evidence that forces are vector quantities. Students will arrange the scales in two different configurations that hold the ring in place: (i) using opposing pairs of spring scales to maintain equilibrium and (ii) using three spring scales to maintain equilibrium. Analysis of each configuration will involve simple vector addition.

LEARNING OBJECTIVE: Students will be able to recognize that opposing forces must balance for a stationary object to remain in equilibrium. Students will be able to use vector addition to account for the magnitude and direction of a force vector.

**Individual Student Lab Stations**

**(i) using opposing spring scales to maintain equilibrium:**  Hold a ring in position with four spring scales, with each scale flat on the table. Align the scales with the horizontal and vertical lines drawn on the next page, with the ring centered with respect to the origin. Try to pull with different forces on the scales A and B. Record values in the table below for several different cases. Compare the forces in scales A and C and in B and D. What is the conclusion?

**D**



**A**

**B**

**C**

**D**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Case** | **Scale A (N)** | **Scale B (N)** | **Scale C (N)** | **Scale D (N)** |
| **1** |  |  |  |  |
| **2** |  |  |  |  |
| **3** |  |  |  |  |
| **4** |  |  |  |  |
| **5** |  |  |  |  |

Conclusion:

**(ii) using three spring scales to maintain equilibrium:** Replace spring scales **C** and **D** with a single spring scale **E**. The person with scale E should adjust the force and the angle, so the ring does not move.



**A**

**B**

**E**

Try these combinations of forces in scales A and B: A = 5 N, B = 5 N; A = 10 N, B = 5 N; A = 15 N, B = 5 N. Hold the ring over the axis origin on the next page, scale A along the (-)-horizontal axis and scale B along the (-)-vertical axis. Draw a labeled line parallel to E for each of the three cases. Use that line to determine the angle of E relative to the (+)-horizontal line. Record the force of scale E (**Scale E Force**) and the angle of scale E (**Scale E angle**) relative to the (+)-horizontal axis in the table below. The columns “Hypotenuse” and “Triangle angle” will be filled in later.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Triangle** | **Hypotenuse** | **Scale E Force (N)** | **Triangle angle** | **Scale E angle** |
| **5 by 5** |  |  |  |  |
| **5 by 10** |  |  |  |  |
| **5 by 15** |  |  |  |  |

Draw three right triangles below, with the legs as listed:

Vert.

Leg

Horiz.

Leg

Drawing of Triangle

Horiz. Leg = 5 cm

Vert. Leg = 5 cm

Drawing of Triangle

Horiz. Leg = 10 cm

Vert. Leg = 5 cm

Drawing of Triangle

Horiz. Leg = 15 cm

Vert. Leg = 5 cm

For each triangle measure the length of the hypotenuse and the angle the hypotenuse makes with respect to the horizontal axis. Enter these values into the columns “Hypotenuse” and “Triangle angle” in the table above. In each of three cases, compare what you found for the hypotenuse with what you found for the force in scale E and its angle.

Scale E does what scales C and D together did. So how do forces combine?

*Note: The interpretation of experiment (i) is that the forces in each direction must balance if body is in equilibrium. The preferred interpretation of experiment (ii) is that the force E is equivalent to the combination of forces C and D (because it performed the same function as did C and D). That E, the vector sum of forces C and D, is equivalent to them does not depend on equilibrium. We do want to add vector forces in dynamics also. A weaker interpretation of experiment (ii) is that A and B are in equilibrium with E. The combination of experiments (i) and (ii) allows us to see both equilibrium and the vector summation of forces.*

**2.3 Balancing forces in two dimensions**

Students should now have some faith that a force is equivalent to its resolved components. They will now analyze a case, again of three forces maintaining static equilibrium of a central ring, but where two of the forces need to be resolved into perpendicular components in order to demonstrate that the forces balance. The forces are now in the vertical plane, while forces in Activity 2.2 were all in the horizontal plane. In this activity, the forces will be resolved into the obvious horizontal and vertical directions. However, in this activity, one of the spring scales is replaced by a hanging block. Two configurations are considered: (i) spring scales are oriented symmetrically about the vertical, and (ii) spring scales are oriented unsymmetrically. Trigonometry and the rules of vector addition are used to demonstrate that the forces must balance.

LEARNING OBJECTIVE: Students will be able to analyze the forces that maintain the static equilibrium of a body. They will be able to use vector addition to demonstrate that the forces on this body must balance.

In rock climbing, cords connected to anchors in the rock hold you up. The cords will be oriented at different angles, which affect the forces. Clamps on oppositely facing chemistry support stands can serve as the anchors to hold the spring scales. Place books on the stand bases if they tend to slip or tip. Use S-hooks to connect the scales to a central ring and to hang the block from the ring.

**Individual Student Lab Stations**





**Block**

**ring**

θ

θ

Record the combined weight of the block, S-hooks, and ring \_\_\_\_\_\_\_\_\_\_\_

**(i) spring scales oriented symmetrically:** Place the support stand clamps at the same height, and attach the scales to the clamps and the ring, so that the scales make the same angle with respect to the horizontal. Record the scale readings and the angle used for three different configurations.

|  |  |  |
| --- | --- | --- |
| **Left scale tension** | **Right scale tension** | **Angle θ with horizontal** |
|  |  |  |
|  |  |  |
|  |  |  |

How do the scale readings change, as the stands are placed farther apart?

On the xy-axis below, draw a vector starting at the origin to represent the scale tension for each trial. The length of each vector should be proportional to the value of tension recorded. And the vector should be positioned to represent accurately the direction of the scale force (*i.e.* the correct angle). Draw a vector starting at the origin representing the weight force.

**(ii) spring scales oriented unsymmetrically:** Now, attach the clamps to the stand at different heights such that each scale makes its own angle with respect to the horizontal axis (θL and θR). Make sure that they are positioned so each scale stretches, but not to its maximum reading.



**Block**

**ring**

θ**R**

θ**L**

**Weight**

**TR**

**TL**

Repeat this multiple times, recording the angles and scale readings in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| **Left tension TL** | **Left angle θL** | **Right tension TR** | **Right angle ΘR** |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Using the measurements, you will calculate the net force in x direction and the y direction on the ring. We can use trigonometry to solve for the horizontal component and vertical component of a force vector. For example,

**D = F cos θ**

**C = F sin θ**

**F**

**C**

θ

**D**

Use these trig functions to resolve each scale force vector (recorded above) into horizontal and vertical components. The scale tension corresponds to the magnitude F, which is different for left and right scales.

For example:

TRx (right scale horizontal) = TR cos θR and TRy (right scale vertical) = TR sin θR

Enter the horizontal and vertical components due to each scale into the table.

Next, determine the total horizontal force and the total vertical force.

Let x be positive if to right, negative if to left. Let y be positive if up, negative if down.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Horizontal Components** | | | **Vertical Components** | | | |
| **TRx** | **TLx** | **TRv - TLx** | **TRv** | **TLy** | **W** | **TRv +TLv - W** |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |

What trend(s) do you notice with the total horizontal force?

What trend(s) do you notice with the total vertical force?

**2.4 Forces experienced by a body on a ramp**

Students will analyze static equilibrium for a body on an inclined ramp, again for simplicity, with three forces acting. In this activity, it is now convenient to resolve the components of the forces with respect to an axis system that is aligned with the ramp. Two cases are examined: (i) a spring scale opposes the component of gravity pulling a freely rolling truck down the ramp and (ii) the force of static friction between a stationary block and the ramp opposes the component of gravity pulling a block down the ramp.

LEARNING OBJECTIVE: Students will be able to analyze the forces that maintain static equilibrium of a body on an inclined ramp. They will be able to resolve forces into perpendicular components and show that opposing forces cancel.

**Individual Student Lab Stations**

**(i) a spring scale opposes the component of gravity pulling a truck down the ramp:** Weigh a wooden toy truck.

Weight of truck = \_\_\_\_\_\_\_\_\_\_N\_

Place the truck on a level ramp and add a weight (about 1 kg) to the truck. Then, raise one end of the ramp and use a spring scale, hooked onto the truck and held parallel to the ramp, to keep the truck in place. With the ramp held at three different angles (approximately 30°, 45°, 60°) record the reading on the spring scale.

|  |  |
| --- | --- |
| **Angle** | **Measured Spring Force Parallel to Ramp** |
|  |  |
|  |  |
|  |  |

In order for the truck to remain in place, the force pulling the truck down the inclined plane must be balanced by the upward pull of the spring scale. Here are all the forces on the truck:

Fscale

θ

W

FN

Fscale

θ

To analyze the forces in this situation, draw inclined axes parallel and perpendicular to the ramp (see below). It can be seen in this drawing that the force FN does not affect the balance of forces along the inclined plane. To better understand how W and Fscale balance, we will use trigonometry to find the pull due to gravity along the inclined plane.

W

θ

Fscale

W

Fscale

θ

FN

Notice that when the ramp is on an angle θ, it appears to form a right-triangle with one side along the table’s surface. By turning your head by θ, you can imagine the new y-axis to be oriented perpendicular to the ramp’s surface and the force due to the spring scale to then point along the (+)-horizontal axis. From consideration of the angle θ, one can calculate the component of W along the (-)-horizontal axis: │Fx│= W∙ sinθ. To see if this component does indeed fulfill our expectation, calculate it for each of the three angles recorded above and compare to the scale readings.

|  |  |  |  |
| --- | --- | --- | --- |
| **Angle** | **W (= Truck + Added Weight)** | **W ∙ sinθ** | **Measured Spring Force Parallel to Ramp** |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

**(ii) force due to friction between a block and the incline surface will serve to counter the component of gravity pulling the block down the ramp:** In this experiment, the force due to friction between a block and the incline surface will serve to counter the force pulling the block down the ramp.

Weigh a large block. The magnitude of the block’s weight is equal to the normal force (N) on the block when it rests on a horizontal surface and there are no other vertical forces. Obtain three different fabric samples: gray, flannel and brown.

Weight of large block = \_\_\_\_\_\_\_\_\_\_\_

For each fabric, in turn, lay the fabric on a horizontal surface and place the block on the fabric. Pull horizontally with a spring scale, recording the minimum force needed to cause the block to slip. From this value, calculate the coefficient of static friction (μs)for the block/fabric system.

|  |  |  |  |
| --- | --- | --- | --- |
| **Fabric** | **Force to Initiate (Fs)** | **Normal Force = N** | **μs = Fs /N** |
| **Gray – Trial 1** |  |  |  |
| **Gray – Trial 2** |  |  |  |
| **Gray – Trial 3** |  |  |  |
| **Flannel – Trial 1** |  |  |  |
| **Flannel – Trial 2** |  |  |  |
| **Flannel – Trial 3** |  |  |  |
| **Brown – Trial 1** |  |  |  |
| **Brown – Trial 2** |  |  |  |
| **Brown – Trial 3** |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| **Fabric** | **Gray** | **Flannel** | **Brown** |
| **Average μs** |  |  |  |

Next, place each of the cloths, in turn, on the board and place the large block on the cloth. Tilt the board until the block starts to slip and record the angle of the board where slipping first starts to occur.

|  |  |
| --- | --- |
| **Surface** | **Angle where slipping begins to occur** |
| Gray |  |
| Flannel |  |
| Brown |  |

Here are all the forces on the block:

Ffric

θ

W

FN

Ffric

θ

W

FN

Again, tilt your head by θ. The force of gravity has components along the inclined plane and perpendicular to it.

W

θ

W∙sinθ

W∙cosθ

For the block to remain stationary, Ffric = W∙sinθ, and the normal force FN is balanced by the component of W perpendicular to the ramp’s surface, FN = W∙cosθ.

When the inclined plane is gradually tilted, the block slips at an angle θs. This was measured and depends on the fabric. The friction force has reached the maximum value Ffric,max = μs ∙ FN . So μs = Ffric,max / FN = W∙sinθs / W∙cosθs = sinθs / cosθs. From the measured angle θs, calculate the coefficient of static friction. Compare with the measurements of μs from pulling the block along the fabric.

|  |  |  |  |
| --- | --- | --- | --- |
| **Fabric** | **θs (angle when slipping begins to occur** | **μs = sinθs / cosθs** | **μs (pulling block on fabric)** |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |